MTH 406: Differential geometry of curves and surfaces

Homework VIII: Geodesics and Gauss-Bonnet Theorem

Problems for practice

1. Reading assignment: Read through the proofs of 4.1(iv) and 4.2(ii) from Kristopher Tapp's book cited in the Lesson Plan.

2. Establish all the assertions in 4.1(iii) and 4.3(iv) of the Lesson Plan.

3. Use the Clairaut's Theorem to identify the geodesics on the torus.

4. A *ruled surface* is a parametrized surface $f : U \subset \mathbb{R}^2 \to \mathbb{R}^3$ that has the form

$$f(s,t) = \gamma(s) + tw(s),\ s \in I, t \in \mathbb{R}$$

   where $\gamma : I \to \mathbb{R}^3$ and $w : I \to \mathbb{R}^3$ are space curves. The line L_s which passes through $\gamma(s)$ and is parallel $w(s)$ is called a *ruling.*

   (a) Show that the tangent surface of a regular curve is a ruled surface.

   (b) Show that the hyperboloid $x^2+y^2-z^2-1$, the cylinder, and $z = xy$ are ruled surfaces.

   (c) Show that the rulings of a ruled surface are geodesics.

5. Show that a closed simply-connected surface S with constant curvature $K > 0$ must be homeomorphic to the sphere.